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COS 488

5/5

V.1 Give an asymptotic expression for the number of strings that do not contain the pattern 0000000001. Do the same for 0101010101.

The OGF of the number of strings that do not contain the pattern 0000000001 is

$$B(z) = \frac{1}{z^{10}+1-2z}$$

which is a rational function with the smallest pole at ~0.5005, meaning we can use the rational transfer theorem to find an approximation.

$$[z^N]\frac{f(z)}{g(z)} \sim \frac{(-\beta)f(1/\beta)}{g'(1/\beta)}\beta^N$$

$$= \frac{-\beta^{N+1}}{\frac{10}{\beta^9}-2} \sim 0.5050 \times 1.998^{N+1}$$

The OGF of the number of bitstring that do not contain the pattern 0101010101 is slightly more complicated due to the pattern's autocorrelation polynomial:

$$B(z) = \frac{1+z^2+z^4+z^6+z^8}{z^{10}+(1-2z)(1+z^2+z^4+z^6+z^8)}$$

This is, however, thankfully still a rational function upon which we can apply the rational transfer theorem.

$$1/\beta = 0.5004$$

$$[z^N]\frac{f(z)}{g(z)} \sim \frac{(-\beta)f(1/\beta)}{g'(1/\beta)}\beta^N$$

$$= \frac{1.333}{2.647}\beta^{N+1} \sim 0.5034 \times 1.998^{N+1}$$