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COS 488

Problem: How many ways are there to make change for N cents without using pennies (and only using quarters, nickels, and dimes)?

Solution:

I was really hoping that the "making change for N cents" problem *wouldn't* be on the lecture slides so that I could make it my exam problem, but I can make a variation of it at least.

Here, we can define the class of all partitions of N composed of 5s, 10s, and 25s:

$$P = MSET(Z^5 + Z^{10} + Z^{25})$$

This results in the following generating function via our symbolic transfer theorems:

$$P(z) = \frac{1}{(1-z^5)(1-z^{10})(1-z^{25})}$$

The dominant singularity would be a pole at $z = 1$ with order 3. We can calculate the coefficient (residue) with the form $h_{-3} = \lim_{z \to 1}(1-z)^3 P(z) = \frac{1}{1250}$, giving us the answer

$$[z^N]P(z) \sim \frac{N^2}{1250*2!} = N^2/2500$$