Analytic Combinatorics Web Exercise V.1

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Let $p = 0000000001$. We see that the autocorrelation polynomial for p is 1, so we have

$$S_p(z) = \frac{1}{z^{10} - 2z + 1} := \frac{f}{g}$$

The dominant root of $z^{10} - 2z + 1$ is $\alpha \approx 0.500493$ (a root of order 1), so we have

$$[z^n]S_p(z) \sim \frac{-f(\alpha)}{\alpha g'(\alpha)} \left(\frac{1}{\alpha}\right)^n = \frac{-1}{\alpha(10\alpha^9 - 2)} \left(\frac{1}{\alpha}\right)^n \approx \boxed{1.00896 \cdot 1.99803^n}.$$

Let $q = 0101010101$. Shifting q to the left, we see that q's end matches its beginning for shifts 0, 2, 4, 6, and 8, so the autocorrelation polynomial for q is $1 + z^2 + z^4 + z^6 + z^8$. Thus, we have

$$S_q(z) = \frac{1 + z^2 + z^4 + z^6 + z^8}{z^{10} + (1 - 2z)(1 + z^2 + z^4 + z^6 + z^8)} := \frac{f}{g}.$$

The dominant root of g is $\alpha \approx 0.500369$ (a root of order 1), so we have

$$[z^n]S_q(z) \sim \frac{-f(\alpha)}{\alpha g'(\alpha)} \left(\frac{1}{\alpha}\right)^n \approx \boxed{1.00620 \cdot 1.99853^n}.$$