COS 488: AC week 4 Q1

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We first evaluate the number of strings that do not contain the pattern 0000000001.
If we slide it left over itself, we see that it has autocorrelation polynomial $c_p(z) = 1$.
We then apply the formula on slide 17 of lecture AC05:

$$S_p(z) = \frac{c_p(z)}{z^p + (1-2z)c_p(z)} = \frac{1}{z^{10} - 2z + 1}$$

We then look for the dominant root of the denominator. There are roots at 1, $\approx$.500493, and some complex values, thus we use the root that is about .500493.
The residue is

$$h_{-1} = -\frac{f(z)}{g'(z)} = -\frac{1}{10z^9 - 2}|_{z \approx .500493} \approx .504975$$

Thus we have that the coefficients are

$$[z^N]S_{0000000001}(z) \sim (.504975)/(.500493)(1/.500493)^N \approx 1.00896 * (1.99803)^N$$

We now evaluate the number of strings that do not contain the pattern 0101010101.
If we slide it left over itself, we see that it has autocorrelation polynomial $c_p(z) = 1 + z^2 + z^4 + z^6 + z^8$.
We then apply the formula on slide 17 of lecture AC05:

$$S_p(z) = \frac{c_p(z)}{z^p + (1-2z)c_p(z)} = \frac{1 + z^2 + z^4 + z^6 + z^8}{z^{10} + (1-2z)(1 + z^2 + z^4 + z^6 + z^8)}$$

$$= \frac{1 + z^2 + z^4 + z^6 + z^8}{z^{10} - 2z^9 + z^8 - 2z^7 + z^6 - 2z^5 + z^4 - 2z^3 + z^2 - 2z + 1}$$

We find the dominant root of the denominator: It has smallest real root at $\approx$ .500369.
To find the residue, we take the derivative of the bottom as above, then plug in our root, giving $h_{-1} = .50347$ Thus we have that the coefficients are

$$[z^N]S_{0101010101}(z) \sim (.50347/500369)(1/.500369)^N \approx 1.006197 * (1.998525)^N$$