COS 488: AC week 4 Q2

Dylan Mavrides

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6/6

Number of objects of size N and the number of parts in a random object of size N for the following classes:
compositions of 1s, 2s, and 3s, triple surjections, and alignments with no singleton cycles.
Compositions of 1s, 2s, and 3s:
We note that a composition of 1s, 2s, and 3s is a sequence of sequences of size 1, 2, or 3. Thus

$$I_{123} = SEQ_1(Z) + SEQ_2(Z) + SEQ_3(Z)$$

$$G(z) = I_{123}(z) = Z + Z^2 + Z^3$$

$$C = SEQ(I)$$

- Number of Objects of size N:
From slide 52, we have

$$[z^N]C(z) \sim \frac{1}{G'(\lambda)\lambda^{N+1}}$$

where $G(\lambda) = \lambda + \lambda^2 + ^3_\lambda = 1$ gives $\lambda \approx .54369$.
Plugging this in to our above equation gives $\approx .3362(1/.5437)^{N+1} = .3362(1.839)^{N+1}$ - Number of Parts in a random object of size N:
(From slide 57) the expected number of components is

$$\mu_N \sim \frac{N+1}{'(\lambda)} + \frac{G''(\lambda)}{G'(\lambda)^2} - 1$$

Plugging in our above λ to the above equation (substituting G for our function as denoted above) gives the answer as .6184N + .2133
Triple Surjections:
Let T be the class of all triple surjections. then

$$T = SEQ(G(Z))$$

$$G(z) = SET_{>2}(Z) = e^Z - Z^2/2 - Z - 1$$

- Number of Objects of size N:
We follow the exact same protocol as above. Setting G(λ) = 1 and solving we get $\lambda \approx 1.568$, plugging this in to the equation given above gives

$$[z^N]T(z) \sim .4486(1/1.568)^{N+1} = .4486(.6378)^{N+1}$$

So then since this is an EGF, we get

$$.4486(.6378)^{N+1}N!$$

(we could use Stirling's approximation to remove the factorial if instructed, but it isn't asked for here) - Number of Parts in a random object of size N:

$$\mu_N \sim \frac{N+1}{'(\lambda)} + \frac{G''(\lambda)}{G'(\lambda)^2} - 1$$

plugging in our new lambda and G(z): Same as above, plugging in to our equation gives: .286N + .050 Alignments with no Singleton Cycles:
Let A be the class of all alignments with no singleton cycles.

$$A = SEQ(CYC_{>1}(Z))$$

$$G(z) = CYC_{>1}(Z) = \ln\frac{1}{1-z} - z$$

We set $G(\lambda) = 1$ and solve for smallest norm root to obtain $\lambda = .8414$.
- Number of Objects of size N:

$$[z^N]A(z) \sim \frac{1}{G'(\lambda)\lambda^{N+1}}$$

plugging in lambda and multiplying by N!, we get our asymptotic approximation:

$$[z^N]A(z)N! = .188495(1.1885)^{N+1}N!$$

- Number of Parts in a random object of size N:

$$\mu_N \sim \frac{N+1}{'(\lambda)} + \frac{G''(\lambda)}{G'(\lambda)^2} - 1$$

plugging in our new lambda and G(z):

$$\mu_N \sim .22403N + .636547$$

Worked with: Eric N, Maryam B.