Homework 10: Exercise V.1

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Pattern: 0000000001

Since no prefix of $p_1 = 0000000001$ is the same as a suffix of p_1, the auto-correlation polynomial for p_1 is $c_{p_1}(z) = 1$. Using the formula from lecture, the generating function $S_{p_1}(z)$ for strings with no occurrences of p is

$$S_{p_1}(z) = \frac{c_{p_1}(z)}{z^{10} + (1-2z)c_{p_1}(z)} = \frac{1}{z^{10} - 2z + 1}$$

Let α be the dominant root of the denominator. A symbolic math package can be used to compute $\alpha = 0.500493$. The coefficients can be extracted via the analytic transfer theorem for meromorphic functions:

$$[z^n]S_{p_1}(z) \sim c\beta^n$$
$$\beta = \frac{1}{\alpha} = 1.9980300$$
$$c = \frac{h_{-1}}{\alpha} = \frac{-1}{\alpha}\frac{f(\alpha)}{g'(\alpha)} = 1.00896$$

Pattern: 0101010101

The auto-correlation polynomial for the pattern $p_2 = 0101010101$ is $c_{p_2}(z) = z^0 + z^2 + z^4 + z^6 + z^8$, since the prefixes and suffixes of p_2 of length $0, 2, 4, 6, 8$ are the same. The generating function is thus

$$S_{p_2}(z) = \frac{c_{p_2}(z)}{z^{10} + (1-2z)c_{p_2}(z)} = \frac{1 + z^2 + z^4 + z^6 + z^8}{z^{10} + (1-2z)(1 + z^2 + z^4 + z^6 + z^8)}$$

Again, let α be the dominant root of the denominator. A symbolic math package can be used to compute $\alpha = 0.500369$. The coefficients can be computed using the analytic transfer theorem for meromorphic GFs:

$$[z^n]S_{p_2}(z) \sim c\beta^n$$
$$\beta = \frac{1}{\alpha} = 1.998525$$
$$c = \frac{h_{-1}}{\alpha} = \frac{-1}{\alpha}\frac{f(\alpha)}{g'(\alpha)} = 1.006197$$