COS 488 - Homework 10 - Web Exercise V.1

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1. Let $p = 0000000001$, so that the autocorrelation polynomial $c_p(z)$ is 1. Then, by the formula given in lecture, the generating function for the class of binary strings that do not contain p is

$$S_p(z) = \frac{c_p(z)}{z^{|p|} + (1-2z)c_p(z)} = \frac{1}{z^{10} - 2z + 1}.$$

Since $S_p(z)$ has a unique closest pole to the origin at $\alpha \approx 0.500493$ with order 1, we have by the AC transfer theorem that

$$\begin{aligned}[z^N]S_p(z) &\sim \frac{1}{2\alpha - 10\alpha^{10}}\left(\frac{1}{\alpha}\right)^N \\ &\approx 1.008956(1.998029)^N.\end{aligned}$$

2. Let $p = 0101010101$, so that the autocorrelation polynomial $c_p(z)$ is $1 + z^2 + z^4 + z^6 + z^8$. Then, by the formula given in lecture, the generating function for the class of binary strings that do not contain p is

$$S_p(z) = \frac{c_p(z)}{z^{|p|} + (1-2z)c_p(z)} = \frac{1 + z^2 + z^4 + z^6 + z^8}{z^{10} - 2z^9 + z^8 - 2z^7 + z^6 - 2z^5 + z^4 - 2z^3 + z^2 - 2z + 1}.$$

Since $S_p(z)$ has a unique closest pole to the origin at $\alpha \approx 0.500369$ with order 1, we have by the AC transfer theorem that

$$\begin{aligned}[z^N]S_p(z) &\sim \frac{1 + \alpha^2 + \alpha^4 + \alpha^6 + \alpha^8}{2\alpha - 2\alpha^2 + 6\alpha^3 - 4\alpha^4 + 10\alpha^5 - 6\alpha^6 + 14\alpha^7 - 8\alpha^8 + 18\alpha^9 - 10\alpha^{10}}\left(\frac{1}{\alpha}\right)^N \\ &\approx 1.006196(1.998525)^N.\end{aligned}$$