COS 488 - Homework 10 - Web Exercise V.2

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1. Let F be the combinatorial class of all compositions of 1's, 2's, and 3's, and let G be the combinatorial class of all 1's, 2's, and 3's, so that we have the construction

$$F = SEQ(G).$$

Since G is a strongly aperiodic supercritical sequence class, $G(z) = z + z^2 + z^3$, and $G(\lambda) = 1$ at the point where $\lambda \approx 0.543689$, we have

$$[z^N]F(z) \sim \frac{1}{\lambda G'(z)} \frac{1}{\lambda^N} \approx 0.618420(1.839287)^N.$$

Moreover, the average number of parts in a random composition of N into 1's, 2's, and 3's is

$$\sim \frac{N}{\lambda G'(\lambda)} \approx 0.618420N.$$

2. Let F be the combinatorial class of all triple surjections, and let G be the combinatorial class of all sets of size at least 3, so that we have the construction

$$F = SEQ(G).$$

Since G is a strongly aperiodic supercritical sequence class, $G(z) = e^z - \frac{z^2}{2} - z - 1$, and $G(\lambda) = 1$ at the point where $\lambda \approx 1.568120$, we have

$$N![z^N]F(z) \sim \frac{N!}{\lambda G'(z)} \frac{1}{\lambda^N} \approx 0.286031(0.637706)^N N!.$$

Moreover, the average number of parts in a random triple surjection of size N is

$$\sim \frac{N}{\lambda G'(\lambda)} \approx 0.286031N.$$

3. Let F be the combinatorial class of all alignments with no singleton cycles, and let G be the combinatorial class of all non-singleton cycles, so that we have the construction

$$F = SEQ(G).$$

Since G is a strongly aperiodic supercritical sequence class, $G(z) = \ln \frac{1}{1-z} - z$, and $G(\lambda) = 1$ at the point where $\lambda \approx 0.841406$, we have

$$N![z^N]F(z) \sim \frac{N!}{\lambda G'(z)} \frac{1}{\lambda^N} \approx 0.224015(1.188487)^N N!.$$

Moreover, the average number of parts in a random alignment of size N with no singleton cycles is

$$\sim \frac{N}{\lambda G'(\lambda)} \approx 0.224015N.$$