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VI.1 Use the standard function scale to directly derive an asymptotic expression for the number of strings in this following CFG: S = E + U×Z×S + D×Z×S, U = Z + U×U×Z, D = Z + D×D×Z.

If we use symbolic transfer to write out the generating functions for *U* and *D*, they end up being the same, which in turn gives us a neat expression for *S*:

$$U(z) = z + zU^2(z)\,,\ D(z) = z + zD^2(z)$$

$$U(z) = D(z) = \frac{1-\sqrt{1-4z^2}}{2z}$$

$$S(z) = 1 + zU(z)S(z) + zD(z)S(z) = 1 + 2zU(z)S(z)$$

$$S(z) = \frac{1}{1-2zU(z)} = \frac{1}{1-\frac{1-\sqrt{1-4z^2}}{1}}$$

$$S(z) = \frac{1}{\sqrt{1-4z^2}}$$

Now apply a change of variables $\alpha = 4z^2$ so that we can use the standard function scale.

$$S(\alpha) = \frac{1}{\sqrt{1-\alpha}} \sim \sum \frac{n^{-1/2}}{\Gamma(1/2)}\alpha^N$$

Change back:

$$S(z) = \sum \frac{1}{\sqrt{\pi n}}(4z^2)^N$$

$$[z^{2n}]S(z) \sim \frac{4^N}{\sqrt{\pi n}}$$