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VI.2 Give an asymptotic expression for the number of rooted ordered trees for which every node has 0, 2, or 3 children. How many bits are necessary to represent such a tree?

Define the class T as the class of all rooted ordered trees for which every node has 0, 2, or 3 children. T is a node attached to some sequence of length 0, 2, or 3 of other such members in this class. Use symbolic transfer to get a generating function:

$$T = Z \times (SEQ_0(T) + SEQ_2(T) + SEQ_3(T))$$

$$T(z) = z[1 + T^2(z) + T^3(z)]$$

Here, we have an invertible tree class. The characteristic function is $\phi(u) = 1 + u^2 + u^3$, which has nonnegative coefficients and is not of an invalid form, is analytic at zero and otherwise satisfies all the conditions from the lecture slides. If we solve the characteristic equation, we get the positive real root:

$$\phi(\lambda) = \lambda\phi'(\lambda)$$

$$1 + \lambda^2 + \lambda^3 = 2\lambda^2 + 3\lambda^3$$

$$2\lambda^3 + \lambda^2 - 1 = 0$$

$$\lambda \sim 0.65730$$

Now apply the transfer theorem for simple varieties of trees to get our coefficient.

$$[z^N]T(z) \sim \frac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}}(\phi'(\lambda))^N N^{-3/2}$$

$$\sim \frac{1}{\sqrt{2\pi(2+6\lambda)/(1+\lambda^2+\lambda^3)}}(2\lambda + 3\lambda^2)^N N^{-3/2}$$

$$\sim 0.2144 \times 2.61072^N N^{-3/2}$$

Not sure how many bits are necessary to encode this… if every bitstring corresponded to exactly one tree, then we'd need the ceiling of log(x) bits where x is our coefficient, but that probably wouldn't be lossless.