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COS 488

VII.1 Use the tree-like schema to develop an asymptotic expression for the number of bracketings with N leaves (see Example I.15 on page 69 and Note VII.19 on page 474)

Define B to be the class of all bracketings with N leaves. A member of B has the construction of either a leaf or a sequence of at least two other members of B, which can be symbolically transferred to an OGF quite easily:

$$B = Z + SEQ_{\geq 2}(B)$$

$$B(z) = z + \tfrac{1}{1-B(z)} - 1 - B(z)$$

This is a tree-like class, as we have a generating function of the form $F(z) = \phi(z, F(z))$. It has the characteristic function $\phi(z,w) = z + \tfrac{1}{1-w} - 1 - w$, is aperiodic and smooth-implicit. Now, to solve its characteristic system:

$$\phi(r,s) = r + \tfrac{1}{1-s} - 1 - s = s$$

$$\phi_w(r,s) = \tfrac{1}{(1-s)^2} - 1 = 1$$

The second equation gives us $s = 1 - \tfrac{1}{\sqrt{2}}$, and plugging in that value for s into the first equation gives us $r = 3 - 2\sqrt{2}$. From here, we can use the transfer theorem from the lecture slides to solve for our coefficient:

$$[z^N]B(z) = \tfrac{\alpha}{2\sqrt{\pi}}(\tfrac{1}{r})^N N^{-3/2}$$

$$\alpha = \sqrt{\tfrac{2r\phi_z(r,s)}{\phi_{ww}(r,s)}} = \sqrt{\tfrac{2r(1)}{(\frac{-2}{(s-1)^3})}} \sim 0.246293$$

$$[z^N]B(z) \sim 0.069477932423 \times 5.82842712474619^N N^{-3/2}$$