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Problem: How many labelled 3-regular graphs are there of N elements? And how many components are there on average in a random graph of this sort?

Solution: Let's start by defining a class R, the class of all 3-regular graphs. Define its construction as the set of undirected cycles with length greater than 3. Symbolic transfer gives us the following generating equation:

$$R(z) = \exp(\tfrac{1}{2}\ln\tfrac{1}{1-z} - \tfrac{z}{2} - \tfrac{z^2}{4} - \tfrac{z^3}{6})$$

This generating function is in exp-log form, with a singularity at 1 being closest to the origin. Thus we can easily use the transfer theorem from lecture 6 to find our answer and average number of components.

$$G(z) \sim \alpha \log \tfrac{1}{1-z/\rho} + \beta$$

where α=1/2, β=11/12, ϱ=1

$$[z^N]R(z) \sim \tfrac{e^{\beta}}{\Gamma(\alpha)}(\tfrac{1}{\rho})^N N^{1-\alpha}$$

$$\sim \frac{e^{\frac{11}{12}}}{\sqrt{\pi N}}$$

So, our number of 3-regular graphs with N components is asymptotically equal to $N!\frac{e^{\frac{11}{12}}}{\sqrt{\pi N}}$ and the average number of components in such a graph is roughly $\frac{1}{2}\ln N$