COS 488: AC week 5 Q1

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We begin with the following CFG:

$$S = E + U \times Z \times S + D \times Z \times S$$
$$U = Z + U \times U \times Z$$
$$D = Z + D \times D \times Z$$

or as a function of z:

$$S(z) = 1 + zS(z)(U(z) + D(z)) \iff S(z) = \frac{1}{1 - z(U(z) + D(z))}$$

and using the quadratic equation we get

$$U(z) = D(z) = \frac{1 - \sqrt{1 - 4z^2}}{2z}$$

so substituting we get

$$S(z) = \frac{1}{\sqrt{1 - 4z^2}} = (1 - 4z^2)^{-1/2}$$

We now find the asymptotics for a similar generating function

$$A(z) = (1 - z)^{-1/2}$$

using the standard function scale transfer theorem with $\alpha = -1/2$. This gives the asymptotic approximation:

$$[z^n]A(z) \sim \frac{n^{-1/2}}{\sqrt{\pi}}$$

Now we recall that $A(z^2)$ makes every other coefficient of z^n alternate between 0 and the originally ordered terms, thus we obtain the asymptotic approximation for $A(z^2)$:

$$[z^{2n}]A(z^2) \sim \frac{(n/2)^{-1/2}}{\sqrt{\pi}}$$

(- and 0 if n is odd). Finally we obtain the asymptotic approximation for $A(4(z^2)) = A((2z)^2) = S(z)$ by applying the usual term-operation to our existing approximation - i.e. the coefficients of the terms are multiplied by $2^{2n} = 4^n$ (for the coefficient of z^{2n}):

$$[z^{2n}]S(z) \sim 4^n \frac{(n)^{-1/2}}{\sqrt{\pi}} = \frac{4^n}{\sqrt{\pi n}}$$

and 0 if the power of z is odd.