COS 488: AC week 5 Q2

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May 4, 2017

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The desired class is defined by:

$$F = Z \times SEQ_{0,2,3}(F)$$

We now use the idea of invertible tree classes from slide 40 of AC06 to obtain our asymptotic approximation for F(z).
We first prove that $F(z) = z\phi(F(z)) = zSEQ_{0,2,3}(z)$ is λ-invertible. Note that $\phi(u) = 1 + u^2 + u^3$ has nonnegative coefficients, and is not of the form $\phi_0 + \phi_1 u$. It is analytic at 0 since it's a polynomial with radius of convergence infinity.
The characteristic equation $\phi(\lambda) = \lambda\phi'(\lambda)$ gives $\lambda^3 + \lambda^2 + 1 = 3\lambda^3 + 2\lambda^2$ which has a real root at $\lambda = .6573$. Thus using the theorem we get:

$$[z^n]F(z) \sim \frac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}}(\phi'(\lambda))^n n^{-3/2} = .263(2.61073)^n n^{-3/2}$$

This is our estimation for the number of rooted ordered trees for which every node has 0, 2, or 3 children.
If we establish a bijection (possibly an injection, up to one bit off of a bijection) between these trees and binary numbers, then we need log-base-2 of the number of trees. Thus taking log base 2 (and thus dropping the multiplicative factor, since it only adds a constant, and dropping the $n^{-3/2}$ since it only causes an added order lg(n) term, and ignoring the ceiling since it only affects our expression by an $O(1)$ term, we get that we need $\sim 1.38445n$ bits to represent a tree of this form of size n.
Worked with: Eric N