COS 488: AC week 5 Q3

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From page 69, line 71 in the textbook, we have that the desired class has implicit generating function:

$$S(z) = z + \frac{S(z)^2}{1 - S(z)}$$

We now see on pages 468 and 469 of the textbook, in the Tree-like structures and implicit functions section, we have a function satisfying the smooth implicit function schema. To prove this we see that the above function has the form:

$$S(z) = G(z, S(z))$$

$$G(z, w) = z + \frac{w^2}{1 - w}$$

(I_1) $G(z, w)$ is analytic in $(-\infty, \infty) \times (-1, 1)$ because z is analytic everywhere, and $\frac{1}{1-w}$ is analytic between -1 and 1.
(I_2) Note that there are no constant terms, so $g_{0,0} = 0$, and there is no w term, so $g_{0,1} \neq 1$, and there exist other positive terms (if you take partial derivatives you easily see there will be positive coefficient terms).
(I_3) We see this condition is satisfied by taking G(r,s) and $G_w(r, s)$ and solving the system of quadratic equations for values that lie in our region of analyticity. This gives: $r = 3 - 2\sqrt{2}$ and $s = 1 - 1/sqrt2$.
Thus S(z) converges at z $= 3 - 2\sqrt{2}$ where it has a square-root singularity.
We apply Theorem VII.3 and see that we have:

$$[z^N]S(z) \sim \sqrt{\frac{rG_z(r,s)2\pi N^3}{r}}^{-N} = \sqrt{\frac{3\sqrt{2}-4}{16\pi N^3}}(3+2\sqrt{2})^n \approx .070(5.828)^N N^{-3/2}$$

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