COS 488 - Homework 11 - Web Exercise VI.1

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The construction for $U(z)$ gives

$$U(z) = z + zU(z)^2,$$

so $U(z) = \frac{1-\sqrt{1-4z^2}}{2z}$. Since D has the same construction, $D(z) = \frac{1-\sqrt{1-4z^2}}{2z}$ as well. Therefore,

$$S(z) = 1 + zU(z)S(z) + zD(z)S(z),$$

so

$$S(z) = \frac{1}{1 - z(U(z) + D(z))} = (1 - 4z^2)^{-1/2},$$

so by the standard function scale, we have

$$[z^{2n}]S(z) \sim \frac{4^n}{\sqrt{\pi n}}.$$