# COS 488 - Homework 11 - Web Exercise VI.2

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Let $T$ be the combinatorial class of all rooted ordered trees in which every node has 0, 2, or 3 children. Then, we have the construction

$$T = z \times (E + T \times T + T \times T \times T),$$

which gives the OGF equation

$$T(z) = z(1 + T(z)^2 + T(z)^3).$$

Let $\phi(u) = 1 + u^2 + u^3$, so that $T$ is a simple variety of $\lambda$-invertible trees with OGF equation $T(z) = z\phi(T(z))$ where $\lambda \approx 0.6573$. Therefore, by the transfer theorem for simple varieties of trees, we have

$$[z^n]T(z) \sim \frac{1}{\sqrt{2\pi n^3}} \sqrt{\frac{\phi(\lambda)}{\phi''(\lambda)}} \phi'(\lambda)^n \approx \frac{1}{\sqrt{2\pi n^3}} \sqrt{\frac{1.7160}{5.9438}} (2.6107)^n \approx \frac{0.5373(2.6107)^n}{\sqrt{2\pi n^3}}.$$

Therefore, the number of bits necessary to represent such a tree with $n$ nodes is

$$\sim \lg(2.6107^n) \approx 1.3844n.$$