COS 488 - Homework 11 - Web Exercise VII.1

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5/5

Let S be the class of all bracketings, and let $\Phi(z,w) = z + \frac{1}{1-w} - 1 - w$, so that by the result in lecture we have

$$S(z) = \Phi(z, S(z)).$$

Let $r, s \in \mathbb{R}$ satisfy $\Phi(r,s) = s$ and $\Phi_w(r,s) = 1$. Since $\Phi_w(r,s) = \frac{1}{(1-s)^2} - 1$, we have $s = 1 - \frac{\sqrt{2}}{2}$ and $r = 3 - 2\sqrt{2}$. Therefore, by the transfer theorem for implicit tree-like classes, we have

$$\begin{aligned}
[z^n]S(z) &\sim \sqrt{\frac{r\Phi_z(r,s)}{2\pi\Phi_{ww}(r,s)n^3}}\left(\frac{1}{r}\right)^n \\
&= \sqrt{\frac{r(1)}{2\pi 2/(1-s)^3 n^3}}\left(\frac{1}{r}\right)^n \\
&= \sqrt{\frac{r(1-s)^3}{4\pi n^3}}\left(\frac{1}{r}\right)^n \\
&= \sqrt{\frac{3\sqrt{2}-4}{16\pi n^3}}(3+2\sqrt{2})^n \\
&\approx 0.0695(5.8284)^n n^{-3/2}.
\end{aligned}$$