

# COS 488 Problem Set #11 Question #1

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Since  $\mathcal{U} = \mathcal{Z} + \mathcal{U} \times \mathcal{U} \times \mathcal{Z}$ , and since  $\mathcal{D}$  satisfies the same recurrence, we obtain

$$\begin{aligned}U(z) = D(z) &= \frac{1 - \sqrt{1 - 4z^2}}{2z} \\ \mathcal{S} &= \epsilon + \mathcal{U} \times \mathcal{Z} \times \mathcal{S} + \mathcal{D} \times \mathcal{Z} \times \mathcal{S} \\ S(z) &= 1 + (1 - \sqrt{1 - 4z^2})S(z) \\ S(z) &= (1 - 4z^2)^{-1/2}\end{aligned}$$

By the standard function scale,  $[z^n](1 - z)^{-1/2} \sim \frac{N^{-1/2}}{\Gamma(1/2)} = \frac{1}{\sqrt{\pi N}}$ .

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You need to change variables back from your  $z \rightarrow 4z^2$  substitution to get the original coefficients.

This will give you a  $4^N$  term for  $[z^{2N}]$