COS 488 Problem Set #11 Question #2

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The recurrence for the combinatorial class $\mathcal{T}$ is given by

$$\begin{aligned}\mathcal{T} &= \mathcal{Z} + \mathcal{Z} \times \mathcal{T} \times \mathcal{T} + \mathcal{Z} \times \mathcal{T} \times \mathcal{T} \times \mathcal{T} \\ T(z) &= z(1 + T(z)^2 + T(z)^3)\end{aligned}$$

We can show that this is an invertible tree class with $\phi(z) = 1 + z^2 + z^3$. First of all, ϕ clearly has nonnegative coefficients and is entire. $u\phi'(u) - \phi(u) = 2u^3 + u^2 - 1$, and as a cubic polynomial must have a real root λ. In particular, since this polynomial goes to positive infinity as u increases and it evaluates to -1 at 0, by IVT it has a positive real root $\lambda \approx 0.657298$. Therefore this is an invertible tree class, and by the transfer theorem

$$\begin{aligned}[z^n]T(z) &\sim \frac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}}(\phi'(\lambda))^N N^{-3/2} \\ &= \sqrt{\frac{\lambda^3 + \lambda^2 + 1}{2\pi(2 + 6\lambda)}}(2\lambda + 3\lambda^2)^N N^{-3/2} \\ &\approx 0.214358 \cdot 2.61072^N N^{-3/2}\end{aligned}$$

Bits needed to represent tree?

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