David Luo
Exercise 2.17

3/5

Assume N is very large. Through simplifying the given recurrence, we have the equation

$$A_N = \frac{N-6}{N}A_{N-1} + 2$$

which has a summation factor of

-0.5 This evaluates to 6!/N(N-1)..(N-5) = 1/Nc6

$$\frac{N-6}{N} * \frac{N-7}{N-1} * \frac{N-8}{N-2} * \ldots * \frac{2}{8} * \frac{1}{7} = \frac{1}{7N(N-1)(N-2)(N-3)(N-4)(N-5)}$$ through telescoping.

Dividing both sides of the equation by the summation factor yields a whole bunch of sums:

$$7N(N-1)(N-2)(N-3)(N-4)(N-5)A_N = 7(N-1)(N-2)(N-3)(N-4)(N-5)(N-6)A_{N-1} + 14N(N-1)(N-2)(N-3)(N-4)(N-5)$$

-0.5 There is no +1 term

$$= 1 + 14\sum_{k=1}^{N} k(k-1)(k-2)(k-3)(k-4)(k-5)$$

Note that this stops telescoping at N = 6 since further terms are zeroed out in the product.

$$A_N = \frac{1+14\sum_{k=1}^{N} k(k-1)(k-2)(k-3)(k-4)(k-5)}{7N(N-1)(N-2)(N-3)(N-4)(N-5)} = O\left(\frac{2\sum_{k=1}^{N} k(k-1)(k-2)(k-3)(k-4)(k-5)}{N(N-1)(N-2)(N-3)(N-4)(N-5)}\right)$$ after removing the small term.

The scary numerator can be broken down into a polynomial, which Mathematica says is

$$A_N = O\left(\frac{\frac{2}{7}(N+1)N(N-1)(N-2)(N-3)(N-4)(N-5)}{N(N-1)(N-2)(N-3)(N-4)(N-5)}\right) = \frac{2}{7}(N+1)$$

-0.5 The use of asymptotic notation and Mathematica is incorrect for getting the exact expression for this explicit recurrence.

+0.5 for addressing A_N representing average # of 2-nodes (bonus)

So on average, the number of 2-nodes is a bit more than $\frac{2}{7}$ of the nodes in the 2-3 tree.

-1 What are the values of A_N for N < 6?

As a side note while Mathematica is an excellent tool for sanity checking/ verification I would advise against using such solvers as a means of solution. Unless otherwise stated no problem in the course will ever require the use of such computational tools.