3.5/5

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Exercise 3.28

The expression $[z^n]\frac{1}{\sqrt{1-z}}ln\frac{1}{1-z}$ is the derivative, with respect to α, of $[z^n](1-z)^{-\alpha}$ when $\alpha = \frac{1}{2}$.

Expanding the second expression gives us

$$1 + \alpha z + \frac{\alpha(\alpha+1)}{2}z^2 + \frac{\alpha(\alpha+1)(\alpha+2)}{6}z^3 + \ldots = \sum_{k\geq 0}\binom{\alpha}{k}z^k$$

-1 pt, should be (a+k-1)Ck

And differentiating it term-by-term with respect to α yields

$$0 + z + \frac{2\alpha+1}{2}z^2 + \frac{3\alpha^2+6\alpha+2}{6}z^3 + \ldots = \sum_{k\geq 0}\binom{\alpha}{k}(H_\alpha - H_{\alpha-k})z^k$$

-0.5 pt, show your work more explicitly--I think you would have caught your previous error this way. Should be H_(alpha+k-1) - H_alpha-1) but consistent w/ previous mistake

Where H_α is the α^{th} Harmonic number. Plug in $\alpha = \frac{1}{2}$, and we have

$$[z^n]\frac{1}{\sqrt{1-z}}ln\frac{1}{1-z} = \binom{\frac{1}{2}}{n}(H_{\frac{1}{2}} - H_{\frac{1}{2}-n})$$

You made an indexing error at the start and the final form is a little minimalist, but this is more or less correct. A possible form for the correct answer is (n + 0.5)Cn*(H_{n - 0.5} - H_{-0.5}), where we interpret the parenthetical as sum_{1 =< i =< n} 1/(i - 0.5). That said, I'd prefer if you wrote it as (2nCn)/4^n*(2H_{2n} - H_n), especially because H_a is a little meaningless unless a is a nonnegative integer.