COS 488
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PS2 - Q2

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We first expand and combine terms to rearrange the equation:

$$A_N = \frac{N-6}{N} A_{N-1} + 2 \qquad \text{for } N > 1$$

Now, if $N > 6$, we can divide equation by the summation factor:

$$\frac{N-6}{N}\frac{N-7}{N-1}\dots\frac{1}{7} = \frac{(N-6)!6!}{N!} = \frac{1}{\binom{N}{6}} \qquad \text{for } N > 6$$

which gives:

$$\binom{N}{6} A_N = \frac{(N-7)!6!}{(N-1)!} A_{N-1} + 2\binom{N}{6} = \binom{N-1}{6} A_{N-1} + 2\binom{N}{6} \qquad \text{for } N > 6$$

Telescoping the equation gives:

$$\binom{N}{6} A_N = 2\sum_{7 \le j \le N} \binom{j}{6} + \binom{6}{6} A_6 \qquad \text{for } N > 6$$

Using the upper binomial sum formula, we get:

$$2\sum_{7 \le j \le N} \binom{j}{6} + \binom{6}{6} A_6 = 2(\sum_{0 \le j \le N} \binom{j}{6} - 1) + A_6 = 2\binom{N+1}{7} - 2 + A_6 \qquad \text{for } N > 6$$

Now we can compute A_N for $N \le 6$ explicitly from the first expression:

$$A_1 = 1$$

$$A_2 = \frac{2-6}{2} A_1 + 2 = -2 + 2 = 0$$

$$A_3 = \frac{3-6}{3} A_2 + 2 = 2$$

$$A_4 = \frac{4-6}{4} A_3 + 2 = -1 + 2 = 1$$

$$A_5 = \frac{5-6}{5} A_4 + 2 = -0.2 + 2 = 1.8$$

$$A_6 = \frac{6-6}{6} A_5 + 2 = 2$$

and hence:

$$\binom{N}{6} A_N = 2\binom{N+1}{7} \qquad \text{for } N > 6$$

which gives

$$A_N = \frac{2\binom{N+1}{7}}{\binom{N}{6}} = \frac{2}{7}(N+1) \qquad \text{for } N > 6$$

Note that at $N = 6$ we also have $\frac{2}{7}(6+1) = 2 = A_6$, so we can write:

$$A_N = \frac{2}{7}(N+1) \qquad \text{for } N \geq 6$$

and $A_1, ..., A_5$ as previously calculated.

How does this relate to the average number of 2-nodes after N steps?