A2$_Q$2

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For N ¿ 1 and with $A_1 = 1$ we have:

$$A_N = A_{N-1} - \frac{2A_{N-1}}{N} + 2(1 - \frac{2A_{N-1}}{N})$$

$$A_N = A_{N-1} - \frac{6A_{N-1}}{N} + 2 = A_{N-1}(1 - 6/N) + 2 = A_{N-1}\frac{N-1}{N} + 2$$

$$\implies NA_N = (N-6)A_{N-1} + 2N$$

$$\implies \frac{N(N-1)(N-2)(N-3)(N-4)(N-5)}{6!}A_N$$

$$= \frac{(N-1)(N-2)(N-3)(N-4)(N-5)(N-6)}{6!}A_{N-1} + \frac{2N(N-1)(N-2)(N-3)(N-4)(N-5)}{6!}$$

$$= \binom{n}{6}A_N = \binom{N-1}{6}A_{N-1} + 2\binom{N}{6}$$

$$\implies \binom{N}{6}A_N = \binom{1}{6}A_1 + 2\sum_{k=2}^{N}\binom{N}{6} = 0 + 2\binom{N+1}{7}$$

Divide by N choose 6 and expand the factorial expression for the combinations:

$$A_N = 2\frac{(N+1)!}{(N-6)!7!}\frac{6!(N-6)!}{N!} = \frac{2(N+1)}{7}.$$

-1 What if N < 6?

Also, how does this relate to the average number of 2-nodes after N steps?