Homework 2: Exercise 2.13

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5/5

Multiplying both sides of the recurrence by $n+1$ gives a telescoping sum:

$$\begin{aligned}
a_n &= \frac{n}{n+1}a_{n-1} + 1 \\
(n+1)a_n &= (n+1) + na_{n-1} \\
(n+1)a_n &= (n+1) + n + \cdots + 2a_1 \\
(n+1)a_n &= (n+1) + n + \cdots + 2 + a_0 \\
(n+1)a_n &= \sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2} \\
a_n &= \frac{n+2}{2}.
\end{aligned}$$

We can verify this closed form by induction:

$$a_n = \frac{n}{n+1} \cdot a_{n-1} + 1 = \frac{n}{n+1} \cdot \frac{(n-1)+2}{2} + 1 = \frac{n}{2} + 1 = \frac{n+2}{2}.$$