Homework 2: Exercise 2.17

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Simple algebra gives

$$A_N = A_{N-1} - \frac{2A_{N-1}}{N} + 2(1 - \frac{2A_{N-1}}{N})$$
$$A_N = A_{N-1}(1 - \frac{2}{N} - \frac{4}{N}) + 2$$
$$A_N = \frac{N-6}{N}A_{N-1} + 2$$
$$N(N-1)\cdots(N-5)A_N = (N-1)(N-2)\cdots(N-6)A_{N-1} + 2N(N-1)\cdots(N-5)$$
$$\binom{N}{6}A_N = \binom{N-1}{6}A_{N-1} + 2\binom{N}{6}$$
$$\binom{N}{6}A_N = \binom{1}{6}A_1 + 2\left[\binom{N}{6} + \cdots + \binom{2}{6}\right]$$
$$\binom{N}{6}A_N = 2\sum_{k=1}^{N}\binom{k}{6} = 2\binom{N+1}{7}$$
$$A_N = 2\frac{\binom{N+1}{7}}{\binom{N}{6}} = \frac{2(N+1)}{7}.$$

This can be confirmed by induction:

$$A_N = \frac{N-6}{N}A_{N-1} + 2 = \frac{N-6}{N}\cdot 2\cdot\frac{(N-1)+1}{7} + 2 = 2\cdot\frac{N-6}{7} + 2 = \frac{2(N+1)}{7}.$$

-1 What if N < 6?

Also, how does this relate to the average number of 2-nodes after N steps?