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COS 488/MAT 474
Problem Set 2, Q1

5/5

AofA Exercise 2.13 Solve the recurrence

$$a_n = \frac{n}{n+1} a_{n-1} + 1$$

for $n > 0$ with $a_0 = 1$.

Solution. For all $N \geq 1$, we have

$$\begin{aligned}
(n+1)a_N &= na_{n-1} + (n+1) && \text{(multiply by } n+1\text{)} \\
&= \sum_{k=1}^{n+1} k \\
&= \frac{(n+1)(n+2)}{2} \\
\implies a_n &= \frac{n+2}{2}.
\end{aligned}$$

Therefore, we have $a_n = \frac{n}{2} + 1$ for all $n \geq 0$.