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COS 488 - Homework 2 - Question 1

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Let $a_n = \frac{n}{n+1}a_{n-1} + 1$ for all $n > 0$, and let $a_0 = 1$. When we divide this recurrence by the summation factor

$$\frac{n}{n+1}\frac{n-1}{n}\cdots\frac{1}{2} = \frac{1}{n+1},$$

we get $(n+1)a_n = na_{n-1} + (n+1)$, which telescopes as

$$(n+1)a_n = na_{n-1} + (n+1) = (n-1)a_{n-2} + n + (n+1) = a_0 + \sum_{k=2}^{n+1} k = 1 + \frac{(n+1)(n+2)}{2} - 1 = \frac{(n+1)(n+2)}{2},$$

so $a_n = \frac{n+2}{2}$ for all $n > 0$ (and indeed for all $n \geq 0$).