COS 488 Problem Set #2 Problem #2

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See below

Here is my analysis for the recurrence relation as stated:

$$\begin{aligned} A_N &= A_{N-1} - \frac{2A_{N-1}}{N} + 2\left(1 - \frac{2A_{N-1}}{N}\right) \\ &= A_{N-1}\left(1 - \frac{6}{N}\right) + 2 \\ NA_N &= A_{N-1}(N-6) + 2N \\ \binom{N}{6}A_N &= \binom{N-1}{6}A_{N-1} + 2\binom{N}{6} \end{aligned}$$

Expanding out the recurrence we obtain

$$\binom{N}{6}A_N = A_6 + 2\sum_{M=7}^{N}\binom{M}{6}$$

Note $6A_6 = A_{6-1}(6-6) + 12 \implies A_6 = 2$. Moreover, $\sum_{M=7}^{N}\binom{M}{6} = \sum_{M=6}^{N}\binom{M}{6} - 1 = \binom{N+1}{7} - 1$. Hence, for $N > 6$,

$$\begin{aligned} \binom{N}{6}A_N &= 2\binom{N+1}{7} \\ A_N &= \frac{2(N+1)}{7} \end{aligned}$$

It remains to compute the finitely many remaining values:

N	A_N
0	0
1	2
2	-2
3	4
4	0
5	2

Initial condition given is A_1 = 1 (corrected through both the errata and on the course website), though this is correct for A_0 = 0 as given in the book. It's best to check the errata and use the course site for the most updated version of the problem, (in the future this will count as a minor error)

How does this relate to the average number of 2-nodes after N steps?