

David Luo  
Exercise 4.9

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If  $\alpha < \beta$ , then  $\lim_{N \rightarrow \infty} \frac{\alpha^N}{\beta^N} = \lim_{N \rightarrow \infty} \left(\frac{\alpha}{\beta}\right)^N = 0$

-1pt, this shows  $\alpha^n = o(\beta^n)$  but that doesn't really show "exponentially small," e.g.  $1 = o(n)$  but  $1/n$  isn't an exponentially small sequence

Let  $\alpha = 1.1$ ,  $\beta = 1.2$

For  $N = 10$ , absolute error =  $\alpha^N = 1.1^{10} = 2.594$ , relative error =  $\frac{\alpha^N}{\beta^N} = \left(\frac{1.1}{1.2}\right)^{10} = 0.419$

For  $N = 100$ , absolute error =  $\alpha^N = 1.1^{100} = 13780.612$ , relative error =  $\frac{\alpha^N}{\beta^N} = \left(\frac{1.1}{1.2}\right)^{100} = 0.000166\dots$

-1pt, relative error is  $\frac{\alpha^n}{\alpha^n + \beta^n}$