

Use Stirling's Approximation:

$$(3N)! / (N!)^3 = \sqrt{6\pi N} \left(\frac{3N}{e}\right)^{3N} / \left[\sqrt{2\pi N} \left(\frac{N}{e}\right)^{N^3}\right]^3$$

This expression here is sufficient if error term is included

$$= \frac{\sqrt{6\pi N} \left(\frac{3N}{e}\right)^{3N}}{2\pi N \sqrt{2\pi N} \left(\frac{N}{e}\right)^{3N}} = \frac{\sqrt{3}}{2\pi N} (3)^{3N}$$

$$= \frac{\sqrt{3}}{2\pi N} \left(1 + N \log(27) + \frac{1}{2} N^2 \log^2(27) + \frac{1}{6} N^3 \log^3(27) + O(N^4)\right)$$

$$= N^{-1} \frac{\sqrt{3}}{2\pi} + \frac{\sqrt{3}}{2\pi} \log(27) + N \frac{\sqrt{3}}{4\pi} \log^2(27) + N^2 \frac{\sqrt{3}}{12\pi} \log^3(27) + O(N^3)$$

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What about the $(1+O(1/N))$ terms?

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I don't follow this equality. How are you applying the Taylor series expansion? If it's using e^{3N} you dropped the multiplicative $(3/e)^{3N}$ term