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Exercise 4.23

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$$\frac{N}{N-1}ln\frac{N}{N-1} = \frac{N}{N-1}(lnN - ln(N-1))$$

$$= \frac{N}{N-1}(lnN - (lnN + ln(1-\frac{1}{N}))) = \frac{N}{N-1}(-ln(1-\frac{1}{N}))$$

$$= \frac{N}{1-N}(ln(1-\frac{1}{N}))$$

Now just use known expansions:

$$\frac{N}{1-N}ln(1-\frac{1}{N}) = \frac{N}{1-N}\sum_{k\geq 1}(-1)(\frac{1}{kN^k}) = \frac{N}{1-N}(\frac{-1}{N} + \frac{-1}{2N^2} + \frac{-1}{3N^3} + O(\frac{1}{N^4}))$$

-1

geometric series

$$= \frac{N}{N-1}(\frac{1}{N} + \frac{1}{2N^2} + \frac{1}{3N^3} + O(\frac{1}{N^4}))$$

N/N-1 = 1/(1-1/N) = 1 + 1/N + 1/N^2 +...

$$= \frac{1}{N-1} + \frac{1}{2N^2-2N} + \frac{1}{3N^3-3N^2} + O(\frac{1}{N^4})$$

-1

This final form upon partial decomposition does not give a good asymptotic expansion since there are 1/N, 1/N^2 and 1/N^3 terms in the O(1/N^4) you've listed