5/5

Analytic Combinatorics Homework 3 Problem 1

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2/21/2017

We claim that e^{-kN} is exponentially small for any positive k. Suppose for contradiction that $e^{-kN} \neq O\left(\frac{1}{N^M}\right)$, for some positive M. Then $e^{-N} = (e^{-kN})^{\frac{1}{k}} \neq O\left(\left(\frac{1}{N^M}\right)^{\frac{1}{k}}\right) = O\left(\frac{1}{N^{M/k}}\right)$. But we know this to not be the case, since e^{-N} is exponentially small.

We have $\frac{\alpha^N}{\beta^N} = \left(\frac{\alpha}{\beta}\right)^N = e^{N \ln \frac{\alpha}{\beta}} = e^{-N(\ln \beta - \ln \alpha)}$. If $\alpha < \beta$ then $\ln \beta - \ln \alpha > 0$, so $\frac{\alpha^N}{\beta^N}$ is exponentially small, meaning that α^N is exponentially small relative to β^N.

If $\alpha = 1.1$ and $\beta = 1.2$, we have:

N	α^n	β^n	$\alpha^n + \beta^n$	Absolute error	Relative error
10	2.5937	6.1917	8.7855	2.5937	0.2952
100	13780.6123	82817974.52	82831755.13	13780.6123	0.0001664

(These numbers were obtained from the formulas for absolute error, $|\text{val} - \text{val}_{\text{approx}}|$, and relative error, $\left|1 - \frac{\text{val}_{\text{approx}}}{\text{val}}\right|$.)