

Analytic Combinatorics Homework 3 Problem 3

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We have

$$\begin{aligned}\frac{(3N)!}{(N!)^3} &= \exp\left(\ln \frac{(3N)!}{(N!)^3}\right) = \exp(\ln(3N)! - 3 \ln(N!)) \\ &= \exp\left(3N \ln(3N) - 3N + \ln \sqrt{6\pi N} - 3N \ln(N) + 3N - 3 \ln \sqrt{2\pi N} + O\left(\frac{1}{N}\right)\right) \\ &= \exp\left(3N \ln 3 - \ln \frac{2\pi N}{\sqrt{3}} + O\left(\frac{1}{N}\right)\right) = \boxed{\frac{3^{3N+\frac{1}{2}}}{2\pi N} \left(1 + O\left(\frac{1}{N}\right)\right)}.\end{aligned}$$