

Analytic Combinatorics Homework 3 Problem 2

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2/21/2017

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We have

$$\begin{aligned}\frac{N}{N-1} \ln \frac{N}{N-1} &= \frac{1}{1-\frac{1}{N}} \ln \frac{1}{1-\frac{1}{N}} = - \left(1 + \frac{1}{N} + O\left(\frac{1}{N^2}\right) \right) \ln \left(1 - \frac{1}{N} \right) \\ &= \left(1 + \frac{1}{N} + O\left(\frac{1}{N^2}\right) \right) \left(\frac{1}{N} + \frac{1}{2N^2} + O\left(\frac{1}{N^3}\right) \right) \\ &= \frac{1}{N} + \frac{1}{2N^2} + O\left(\frac{1}{N^3}\right) + \frac{1}{N^2} + O\left(\frac{1}{N^3}\right) + O\left(\frac{1}{N^3}\right) \\ &= \boxed{\frac{1}{N} + \frac{3}{2N^2} + O\left(\frac{1}{N^3}\right)}.\end{aligned}$$