COS 488
Eric Wu - ezwu@
Feb 23, 2017

PS3 - Q3 4.5/5

We use the exp-log trick to write

$$\frac{(3N)!}{(N!)^3} = \exp(\ln((3N)!) - 3\ln(N!))$$

Now, using Stirling's approximation $\ln N! = N\ln N - N + \ln\sqrt{2\pi N} + O(\frac{1}{N})$, we get:

$$\exp(\ln((3N)!) - 3\ln(N!))$$

$$= \exp(3N\ln(3N) - 3N + \ln\sqrt{6\pi N} + O(\frac{1}{N}) - 3(N\ln N - N + \ln\sqrt{2\pi N} + O(\frac{1}{N})))$$

$$= \exp(3N\ln 3 + \ln\left(\frac{\sqrt{6\pi N}}{2\pi N\sqrt{2\pi N}}\right) + O(\frac{1}{N}))$$

$$= \exp(3N\ln 3 + \ln\left(\frac{\sqrt{3}}{2\pi N}\right) + O(\frac{1}{N}))$$

$$= \exp(3N\ln 3 + \ln\sqrt{3} - \ln(2\pi N) + O(\frac{1}{N}))$$

-0.5 $$= 9^N * (\sqrt{3}) * (\frac{1}{2\pi N}) * (1 + O(\frac{1}{N}))$$ 3^3N not 3^2N = 9^N?

where in the last equality we have used the fact that $e^{O(1/N)} = 1 + O(1/N)$. Hence,

$$\frac{(3N)!}{(N!)^3} = 9^N\left(\frac{\sqrt{3}}{2\pi N}\right)(1 + O(\frac{1}{N}))$$