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COS 488 Week 3: Q4

Dylan Mavrides

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We look at an arbitrary term:

$$\frac{(N-k)^k(N-k)!}{N!} = exp(ln(N-k)! - ln(N!) + kln(N-k))$$

We apply stirling's approximation twice:

$$= exp((N-k+.5)ln(N-k) - (N-k) + ln\sqrt{2\pi} - (N+.5)lnN + N - ln\sqrt{2\pi} + kln(N-k) + O(1/N))$$

simplifying...

$$= exp((N+.5)ln(1-k/N) + k + O(1/N))$$

We note that

$$ln(1-k/N) = -k/N - k^2/(2N^2) + O(k^3/N^3)$$

thus:

$$= exp(-k - k/2N - k^2/2N - k^2/4N^2 + O(k^3/N^2) + O(k^3/N^3) + k + O(1/N))$$

$$= exp(-k^2/2N + O(k^3/N^2) + O(k/N))$$

We note that this is exactly identical to the stage pre-integration of the Ramanujan Q distribution, which, post-laplace, post-integration, post-euler-maclaurin, ends up at the desired conclusion, that this expression is equal to $\sqrt{\pi N/2} + O(1)$. (We note that the k_0 still works at $o(N^{2/3})$ because if we look at the above, it causes us to get O(1) etc. and the proof goes through.)
(Worked with Maryam B.)