

## COS 488 Week 3: Q2

5/5

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$$\frac{N}{N-1} \ln \frac{N}{N-1} = \frac{1}{1-1/N} (-1) \ln \frac{N-1}{N} = \frac{1}{1-1/N} (-1) \ln(1-1/N)$$

(the log we can expand based on an expansion done in lecture)

$$= \left( \sum_{k=0}^{\infty} \frac{1}{N^k} \right) \left( \sum_{k=1}^{\infty} \frac{1}{kN^k} \right)$$

Which is approximately:

$$(1 + 1/N + 1/N^2 + \dots)(1/N + 1/2N^2 + 1/3N^3) = 1/N + 3/2N^2 + 11/6N^3 + O(1/N^4)$$

(note that we could replace coefficients with harmonic numbers if we want!) (Worked with Maryam B.)