

Homework 3: Exercise 4.38

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5/5

Using the exp-log technique, Stirling's approximation, and simple algebra, we have:

$$\begin{aligned}
 \frac{(3N)!}{(N!)^3} &= \exp(\ln(3N)! - 3 \log(N!)) && \text{[exp-log]} \\
 &= \exp\left(\left(3N + \frac{1}{2}\right) \ln(3N) - 3N + \ln \sqrt{2\pi} + O\left(\frac{1}{N}\right)\right. \\
 &\quad \left. - 3\left(N + \frac{1}{2}\right) \ln N + 3N - 3 \ln \sqrt{2\pi} + O\left(\frac{1}{N}\right)\right) && \text{[Stirling]} \\
 &= \exp\left(\left(3N + \frac{1}{2}\right) \ln 3 - \ln N - 2 \ln \sqrt{2\pi} + O\left(\frac{1}{N}\right)\right) && \text{[collecting terms]} \\
 &= \exp\left(3N \ln 3 + \frac{\ln 3}{2} - \ln N - 2 \ln \sqrt{2\pi} + O\left(\frac{1}{N}\right)\right) && \text{[simplify]} \\
 &= \frac{\sqrt{3}}{2\pi} \cdot \frac{27^N}{N} \left(1 + O\left(\frac{1}{N}\right)\right) && \text{[evaluate exp].}
 \end{aligned}$$