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**Homework 3: Exercise 4.23**

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We will compute the asymptotic expansion for  $\frac{N}{N-1} \ln \frac{N}{N-1}$ .

First, observe that

$$\frac{N}{N-1} = \frac{1}{1 - \frac{1}{N}} = \sum_{k=0}^{\infty} \frac{1}{N^k}$$

as  $N \rightarrow \infty$ .

Additionally, we have

$$\ln \frac{N}{N-1} = -\ln \frac{N-1}{N} = -\ln\left(1 - \frac{1}{N}\right) = \sum_{k=1}^{\infty} \frac{1}{kN^k}.$$

Combining the two terms, we have

$$\begin{aligned} \frac{N}{N-1} \ln \frac{N}{N-1} &= \left( \sum_{k=0}^{\infty} \frac{1}{N^k} \right) \left( \sum_{k=1}^{\infty} \frac{1}{kN^k} \right) \\ &= \left( 1 + \frac{1}{N} + \frac{1}{N^2} + \cdots \right) \left( \frac{1}{N} + \frac{1}{2N^2} + \cdots \right) \\ &= \frac{1}{N} + \frac{1}{2N^2} + \frac{1}{N^2} + O\left(\frac{1}{N^3}\right) \\ &= \frac{1}{N} + \frac{3}{2N^2} + O\left(\frac{1}{N^3}\right). \end{aligned}$$