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COS 488 - Homework 3 - Question 1

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If $\alpha < \beta$, then $\frac{\alpha}{\beta} < 1$, so $\ln \frac{\alpha}{\beta} < 0$. Therefore,

$$\frac{\alpha^n}{\beta^n} = \left(\frac{\alpha}{\beta}\right)^n = e^{n \ln \frac{\alpha}{\beta}}$$

is exponentially small by the discussion in the textbook. Thus, α^n is exponentially small relative to β^n. For $\alpha = 1.1$ and $\beta = 1.2$, we have the following table of absolute and relative errors when $\alpha^n + \beta^n$ is approximated by β^n for $n = 10$ and $n = 100$:

n	absolute	relative
10	2.594	0.2952
100	13780	0.0001664