COS 488 - Homework 3 - Question 2

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We have that

$$\ln\left(\frac{N}{N-1}\right) = -\ln\left(1-\frac{1}{N}\right) = -\left(-\frac{1}{N}-\frac{1}{2N^2}-\frac{1}{3N^3}+O\left(\frac{1}{N^4}\right)\right) = \frac{1}{N}+\frac{1}{2N^2}+\frac{1}{3N^3}+O\left(\frac{1}{N^4}\right)$$

and

$$\frac{N}{N-1} = \frac{1}{1-1/N} = 1+\frac{1}{N}+\frac{1}{N^2}+O\left(\frac{1}{N^3}\right),$$

so

$$\begin{aligned}\frac{N}{N-1}\ln\left(\frac{N}{N-1}\right) &= \left(\frac{1}{N}+\frac{1}{2N^2}+\frac{1}{3N^3}+O\left(\frac{1}{N^4}\right)\right)\left(1+\frac{1}{N}+\frac{1}{N^2}+O\left(\frac{1}{N^3}\right)\right)\\ &= \frac{1}{N}+\frac{3}{2N^2}+\frac{11}{6N^3}+O\left(\frac{1}{N^4}\right).\end{aligned}$$