

COS 488 Problem Set #3 Question #2

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February 23, 2017

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$$\begin{aligned}\frac{N}{N-1} \log \frac{N}{N-1} &= \frac{-\log \frac{N-1}{N}}{\frac{N-1}{N}} \\ &= \frac{-\log \left(1 - \frac{1}{N}\right)}{1 - \frac{1}{N}} \\ &= \left(\sum_{k=1}^{\infty} \frac{N^{-k}}{k} \right) \left(1 + \sum_{k=1}^{\infty} N^{-k} \right) \\ &= \sum_{k=1}^{\infty} N^{-k} \sum_{j=1}^k j^{-1} \\ &= \sum_{k=1}^{\infty} H_k N^{-k} \\ &= \frac{1}{N} + \frac{3}{2N^2} + \frac{11}{6N^3} + O\left(\frac{1}{N^4}\right)\end{aligned}$$