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Exercise 5.1

4.5/5

How many bitstrings of length N have no 000 ?

Using the symbolic method, we are looking for the class of binary strings with no 000, which we will call B . The size of an object in this class is $|b|$, the number of bits in b , made up of 0-bit and 1-bit atoms (both of size 1). We can call the atoms Z_0 and Z_1 .

We are constructing B as either empty or 0 alone or 00 alone, or a 1 followed by a bitstring without 000, or 01 followed by a bitstring without 000, or a 001 followed by a bitstring without 000.

$$B = (\epsilon + Z_0 + Z_0Z_0) + (Z_1 + Z_0Z_1 + Z_0Z_0Z_1) \times B$$

Then we have the OGF equation:

$$B(z) = 1 + z + z^2 + (z + z^2 + z^3)B(z)$$

$$B(z) = \frac{1+z+z^2}{1-z-z^2-z^3}$$

Then apply the rational functions transfer theorem. The largest root of the denominator is roughly 0.54369, so $\beta = 1.83928$ and $\frac{1}{\beta} = 0.54369$. So, for $\frac{f(z)}{g(z)} = B(z)$,

$$[z^N] \frac{f(z)}{g(z)} = \frac{(1.83928)(1+0.54369+(0.54369)^2)}{-1-2(1.83928)-3(1.83928)} (1.83928)^N$$
$$\sim 1.83928^N$$

-0.5

should be $2/\beta - 3/\beta^2$ in denom

You should include the explicit constant