

David Luo
Exercise 5.3

Let U be the set of binary trees with the size of a tree defined to be the total number of nodes (internal plus external), so that the generating function for its counting sequence is $U(z) = z + z^3 + 2z^5 + 5z^7 + 14z^9 + \dots$. Derive an explicit expression for $U(z)$.

We are looking at the class U of all binary trees where size $|t|$ is defined as the number of external and internal nodes in t . We have two atoms, which I will call E and I , denoting external and internal nodes of size 1 each. We construct binary trees as an external node or an internal node connected to two binary trees.

$$U = E + I \times B \times B$$

$$U(z) = z + zU(z)^2$$

Through the quadratic equation,

$$U(z) = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$

-1 pt, it's important that we take the minus sign so that $U(z)$ is well-defined as a OGF, else the function blows up as $z \rightarrow 0$

This is equivalent to the series $z + z^3 + 2z^5 + 5z^7 + 14z^9 + \dots$