

Analytic Combinatorics Homework 4 Problem 1

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Let B be the class of binary strings with no 000. Let Z_0 be the zero bit atom, Z_1 be the one bit atom, and E be the empty string. A binary string with no 000 is empty, 0, 00, or one of 1, 01, and 001 followed by a binary string with no 000. This gives us the construction

$$B = E + Z_0 + Z_0 \times Z_0 + (Z_1 + Z_0 \times Z_1 + Z_0 \times Z_0 \times Z_1) \times B.$$

The corresponding generating function is

$$B(z) = 1 + z + z^2 + (z + z^2 + z^3)B(z),$$

so

$$B(z) = \frac{1 + z + z^2}{1 - z - z^2 - z^3}.$$

Using the formula on slide 7 of the Asymptotics lecture, we set $f(z) = 1 + z + z^2$ and $g(z) = 1 - z - z^2 - z^3$. The largest root of g is approximately 0.544 (multiplicity 1), so we let $\beta \approx \frac{1}{0.544} \approx 1.839$. Then

$$C = \frac{-\beta f\left(\frac{1}{\beta}\right)}{g'\left(\frac{1}{\beta}\right)} \approx 1.137.$$

Thus we have

$$[z^n]B(z) \sim \boxed{1.137 \cdot 1.839^n}.$$