

Analytic Combinatorics Homework 4 Problem 3

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In analogy with slide 37, let Q be the class of permutations whose cycles have odd lengths; then $Q = SET(CYC_{odd}(Z))$ where Z represents a labelled atom and CYC_{odd} is the class of cycles of odd length. The EGF for CYC_{odd} is $z + \frac{z^3}{3} + \frac{z^5}{5} + \dots$, per slide 30. We have

$$\begin{aligned} z + \frac{z^3}{3} + \frac{z^5}{5} + \dots &= \left(z + \frac{z^2}{2} + \frac{z^3}{3} + \dots \right) - \left(\frac{z^2}{2} + \frac{z^4}{4} + \dots \right) \\ &= \ln \frac{1}{1-z} - \frac{1}{2} \left(z^2 + \frac{(z^2)^2}{2} + \frac{(z^2)^3}{3} + \dots \right) \\ &= \ln \frac{1}{1-z} - \frac{1}{2} \ln \frac{1}{1-z^2}. \end{aligned}$$

Thus the EGF for $Q(z)$ is

$$\exp \left(\ln \frac{1}{1-z} - \frac{1}{2} \ln \frac{1}{1-z^2} \right) = \frac{\frac{1}{1-z}}{\left(\frac{1}{1-z^2} \right)^{\frac{1}{2}}} = \boxed{\frac{\sqrt{1-z^2}}{1-z}}.$$

Simplifies to $\text{sqrt}((1+z)/(1-z))$