Analytic Combinatorics Homework 4 Problem 4

Eric Neyman
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Note that the z^N-coefficient of the EGF derived in the previous problem, i.e. $Q(z) = \frac{\sqrt{1-z^2}}{1-z}$, is precisely the probability that a random N-permutation only has cycles of odd length (since it is the number of such permutations divided by $N!$). Write $Q(x) = \frac{\sqrt{(1-z)(1+z)}}{\sqrt{(1-z)(1-z)}} = \frac{\sqrt{1+z}}{\sqrt{1-z}}$. We apply the corollary on slide 45. We have $f(z) = \sqrt{1+z}$, $\rho = 1$, and $\alpha = \frac{1}{2}$. This gives us that the N-th term of the generating function is approximately

$$\frac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} N^{\frac{-1}{2}} = \frac{\sqrt{2}}{\sqrt{\pi N}} = \frac{1}{\sqrt{\frac{\pi N}{2}}},$$

as desired.