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COS 488
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PS4 - Q1

As usual, we can define two atoms, the 0 bit atom and the 1 bit atom represented by Z_0 and Z_1, respectively, which each have GFs equal to z.

Next, we note that a binary string with no 000 is either empty, or 0, or 00, or a 1 or 01 or 001 followed by a binary string with no 000, which gives the construction:

$$B_{000} = E + Z_0 + Z_0 \times Z_0 + (Z_1 + Z_0 \times Z_1 + Z_0 \times Z_0 \times Z_1) \times B_{000}$$

This gives us the OGF equation

$$B_{000}(z) = 1 + z + z^2 + (z + z^2 + z^3)B_{000}(z)$$

Solving for B_{000} gives:

$$B_{000}(z) = \frac{1 + z + z^2}{1 - z - z^2 - z^3}$$

And applying the rational functions transfer theorem gives that the coefficients are asymptotically:

$$[z^N]B_{000}(z) = C\beta^N$$

Where $1/\beta$ is the largest root of g and

$$-\frac{\beta f(1/\beta)}{g'(\beta)}$$

is a constant.

The root of largest modulus in the denominator here, calculated from mathematica, is:

$$1/\beta \approx 1.3562$$

$$\beta \approx 0.737353$$

You actually want the unique root of smallest modulus. This was an error in the slides but corrected on Piazza and under Theorem 4.1 on the website aofa.cs.princeton.edu/40asymptotic/.

And hence we have that

$$[z^N]B_{000}(z) = C \times \beta^N = .40965 * .737353^N$$

Always try to check multiple sources (or try to derive the result or simulate it and sanity check against Wolfram, Mathematica etc) in the future as next time this will be a nontrivial deduction