COS 488
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PS4 - Q3

Here our construction is

$$O = SET(CYC_{odd}(Z))$$

Since we want permutations with only odd cycles, which are precisely sets of odd-cycles of labelled atoms, where Z is the labelled atom with size 1 and GF z. Then we get that the EGF of our construction is:

$$O(z) = \exp\left(\frac{z}{1} + \frac{z^3}{3} + \frac{z^5}{5} + \ldots\right)$$

Now, note that

$$\frac{z}{1} + \frac{z^3}{3} + \frac{z^5}{5} + \ldots = \ln\frac{1}{1-z} - \left(\frac{z^2}{2} + \frac{z^4}{4} + \ldots\right)$$

$$= \ln\frac{1}{1-z} - \frac{1}{2}\left(\frac{z^2}{1} + \frac{z^4}{2} + \ldots\right)$$

$$= \ln\frac{1}{1-z} - \frac{1}{2}\left(\ln\frac{1}{1-z^2}\right)$$

And so substituting that in, we get

$$O(z) = \frac{1}{1-z} * \left(\frac{1}{1-z^2}\right)^{-0.5} = \frac{\sqrt{1-z^2}}{1-z}$$

$$= \sqrt{\frac{1+z}{1-z}}$$

which is our EGF for the number of permutations with cycles of all odd length.